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Hypothesis Testing in the Regression Model

Introduction

This handout extends the handout on "The Multiple Linear Regression model" and refers to its definitions and assumptions in section 2. This handout describes in examples how typical hypothesis are tested.

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1 Small sample, two-sided t-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i is normally distributed and homoskedastic (OLS3a, OLS4a).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_1 = q = 0.06 \quad \text{vs.} \quad H_A : \beta_1 \neq 0.06$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$t = \frac{\widehat{\beta}_1 - q}{\widehat{se}[\widehat{\beta}_1]} = \frac{0.041 - 0.06}{0.011} = -1.727$$

Step 5: Get critical value from t-table, e.g.

$$t_{crit} = t_{1-\alpha/2, N-K-1} = t_{0.975,26} = 2.056$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$|t| = 1.727 < t_{crit} = 2.056$$

$$\Rightarrow \quad H_0 \text{ is not rejected}$$

Step 7: Make concluding statement, e.g.

"The effect of an additional year of education on wage is not significantly different from 6% at the 5% significance level."

2 Small sample, one-sided *t*-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i is normally distributed and homoskedastic (OLS3a, OLS4a).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_1 = q = 0.06 \quad \text{vs.} \quad H_A : \beta_1 < 0.06$$

If $\hat{\beta}_1 \geq 0.06$ immediately go to step 6 and do not reject H_0.

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$t = \frac{\widehat{\beta}_1 - q}{\widehat{se}[\widehat{\beta}_1]} = \frac{0.041 - 0.06}{0.011} = -1.727$$

Step 5: Get critical value from t-table, e.g.

$$t_{crit} = t_{\alpha,N-K-1} = t_{0.05,26} = -t_{1-\alpha,N-K-1} = -t_{0.95,26} = -1.706$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$t = -1.727 < t_{crit} = -1.706$$

$$\Rightarrow \quad H_0 \text{ is rejected}$$

Step 7: Make concluding statement, e.g.

"The effect of an additional year of education on wage is significantly smaller than 6% at the 5% significance level."

3 Small sample, F-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i is normally distributed and homoskedastic (OLS3a, OLS4a).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \quad -\beta_2/(2\beta_3) = 25 \;\Leftrightarrow\; \beta_2 + 50\beta_3 = 0\,, \quad J = 1$$

$$\Rightarrow R\beta = \begin{pmatrix} 0 & 0 & 1 & 50 \end{pmatrix} \beta = q = 0$$

$$H_A : \quad -\beta_2/(2\beta_3) = 25 \;\Leftrightarrow\; \beta_2 + 50\beta_3 \neq 0$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$F = \frac{1}{J}\left(R\widehat{\beta} - q\right)' \left(R\widehat{V}[\widehat{\beta}|X]R'\right)^{-1} \left(R\widehat{\beta} - q\right) = 3.21$$

Step 5: Get critical value from F-table, e.g.

$$F_{crit} = F_{1-\alpha,J,N-K-1} = F_{0.95,1,26} = 4.23$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$F = 3.21 < F_{crit} = 4.23$$

$$\Rightarrow \quad H_0 \text{ is not rejected}$$

Step 7: Make concluding statement, e.g.

"The optimal work experience is not significantly different from 25."

4 Small sample, joint F-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i is normally distributed and homoskedastic (OLS3a, OLS4a).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_2 = 0 \quad \text{and} \quad \beta_3 = 0\,, \quad J = 2$$

$$\Rightarrow R\beta = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \beta = q = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$H_A : \beta_2 \neq 0 \quad \text{and/or} \quad \beta_3 \neq 0$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$F = \frac{1}{J}\left(R\widehat{\beta} - q\right)' \left(R\widehat{V}[\widehat{\beta}|X]R'\right)^{-1} \left(R\widehat{\beta} - q\right) = 5.78$$

Step 5: Get critical value from F-table, e.g.

$$F_{crit} = F_{1-\alpha, J, N-K-1} = F_{0.95,2,26} = 3.37$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$F = 5.78 > F_{crit} = 3.37$$

$$\Rightarrow \quad H_0 \text{ is rejected}$$

Step 7: Make concluding statement, e.g.

"Work experience has a significant effect on wages at the 5% level."

5 Large sample, two-sided *z*-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i may be heteroskedastic (OLS3d, OLS4b).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_1 = q = 0.06 \quad \text{vs.} \quad H_A : \beta_1 \neq 0.06$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$z = \frac{\widehat{\beta}_1 - q}{\widehat{se}[\widehat{\beta}_1]} = \frac{0.041 - 0.06}{0.012} = -1.583$$

Note: $\widehat{se}[\widehat{\beta}_1]$ can be heteroscedasticity- or cluster-robust.

Step 5: Get critical value from standard normal table, e.g.

$$z_{crit} = z_{1-\alpha/2} = z_{0.975} = 1.960$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$|z| = 1.583 < z_{crit} = 1.960$$

$$\Rightarrow \quad H_0 \text{ is not rejected}$$

Step 7: Make concluding statement, e.g.

"The effect of an additional year of education on wage is not significantly different from 6% at the 5% significance level."

6 Large sample, one-sided *z*-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i may be heteroskedastic (OLS3d, OLS4b).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_1 = q = 0.06 \quad \text{vs.} \quad H_A : \beta_1 < 0.06$$

If $\hat{\beta}_1 \geq 0.06$ immediately go to step 6 and do not reject H_0.

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$z = \frac{\widehat{\beta}_1 - q}{\widehat{se}[\widehat{\beta}_1]} = \frac{0.041 - 0.06}{0.012} = -1.583$$

Note: $\widehat{se}[\widehat{\beta}_1]$ can be heteroscedasticity- or cluster-robust.

Step 5: Get critical value from standard normal table, e.g.

$$z_{crit} = z_\alpha = z_{0.05} = -z_{0.95} = -1.645$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$z = -1.583 > z_{crit} = -1.645$$

$$\Rightarrow \quad H_0 \text{ is not rejected}$$

Step 7: Make concluding statement, e.g.

"The effect of an additional year of education on wage is not significantly smaller than 6% at the 5% significance level."

7 Large sample, Wald-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i may be heteroskedastic (OLS3d, OLS4b).

Step 2: Formulate the hypothesis, e.g.

$$H_0: \quad -\beta_2/(2\beta_3) = 25 \;\Rightarrow\; \beta_2 + 50\beta_3 = 0\,, \quad J = 1$$

$$\Rightarrow R\beta = \begin{pmatrix} 0 & 0 & 1 & 50 \end{pmatrix} \beta = q = 0$$

$$H_A: \quad -\beta_2/(2\beta_3) \neq 25$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$W = \left(R\widehat{\beta} - q\right)' \left(R\widehat{Avar}[\widehat{\beta}]R'\right)^{-1} \left(R\widehat{\beta} - q\right) = J \cdot F = 3.07$$

Note: $\widehat{Avar}[\widehat{\beta}_1]$ can be heteroscedasticity- or cluster-robust.

Step 5: Get critical value from χ^2-table, e.g.

$$\chi^2_{crit} = \chi^2_{1-\alpha,J} = \chi^2_{0.95,1} = 3.84$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$W = 3.07 < \chi^2_{crit} = 3.84$$

$$\Rightarrow \quad H_0 \text{ is not rejected}$$

Step 7: Make concluding statement, e.g.

"The optimal work experience is not significantly different from 25."

8 Large sample, joint Wald-Test

Step 1: Introduce the econometric model and parameters, e.g.

Observed is a random sample (OLS2) of $N = 30$ workers i following

$$log(wage_i) = \beta_0 + \beta_1 edu_i + \beta_2 exper_i + \beta_3 exper_i^2 + u_i$$

where the $K = 3$ explanatory variables are exogenous and the error term u_i may be heteroskedastic (OLS3d, OLS4b).

Step 2: Formulate the hypothesis, e.g.

$$H_0 : \beta_2 = 0 \quad \text{and} \quad \beta_3 = 0\,, \quad J = 2$$
$$\Rightarrow R\beta = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \beta = q = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$H_A : \beta_2 \neq 0 \quad \text{and/or} \quad \beta_3 \neq 0$$

Step 3: Set the significance level. E.g.

$$\alpha = 0.05 = 5\%$$

Step 4: Calculate the test statistic, e.g.

$$W = \left(R\widehat{\beta} - q\right)' \left(R\widehat{Avar}[\widehat{\beta}]R'\right)^{-1} \left(R\widehat{\beta} - q\right) = J \cdot F = 11.23$$

Note: $\widehat{Avar}[\widehat{\beta}_1]$ can be heteroscedasticity- or cluster-robust.

Step 5: Get critical value from χ^2-table, e.g.

$$\chi^2_{crit} = \chi^2_{1-\alpha,J} = \chi^2_{0.95,2} = 5.99$$

Step 6: Reject H_0 or do not reject H_0, e.g.

$$W = 11.23 > \chi^2_{crit} = 5.99$$
$$\Rightarrow \quad H_0 \text{ is rejected}$$

Step 7: Make concluding statement, e.g.

"Work experience has a significant effect on wages at the 5% level."

Standard Normal Distribution

The table reports $\Phi(x) = Pr(X \leq x) = \int_{s=-\infty}^{x} \phi(s)ds$, where $\phi(s) = \frac{1}{\sqrt{2\pi}} e^{-0.5s^2}$.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

t-Distribution

The table reports x given $P = Pr(X \leq x)$ and the degrees of freedom ν.

	P						
ν	0.9	0.95	0.975	0.99	0.995	0.999	0.9995
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.920	4.303	6.965	9.925	22.328	31.600
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.091	3.291

Chi-squared Distribution

The table reports x given $P = Pr(X \leq x)$ and the degrees of freedom ν.

	P								
ν	0.005	0.01	0.025	0.05	0.950	0.975	0.990	0.995	0.999
1	0.000039	0.00016	0.00098	0.0039	3.841	5.024	6.635	7.879	10.828
2	0.010003	0.02010	0.05064	0.1026	5.991	7.378	9.210	10.597	13.816
3	0.07172	0.1148	0.2158	0.3518	7.815	9.348	11.345	12.838	16.266
4	0.2070	0.2971	0.4844	0.7107	9.488	11.143	13.277	14.860	18.467
5	0.4117	0.5543	0.8312	1.145	11.070	12.832	15.086	16.750	20.515
6	0.6757	0.8721	1.237	1.635	12.592	14.449	16.812	18.548	22.458
7	0.9893	1.239	1.690	2.167	14.067	16.013	18.475	20.278	24.322
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955	26.124
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589	27.877
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188	29.588
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757	31.264
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300	32.909
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819	34.528
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319	36.123
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801	37.697
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267	39.252
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718	40.790
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156	42.312
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582	43.820
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997	45.315
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401	46.797
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796	48.268
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181	49.728
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.559	51.179
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928	52.620
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290	54.052
27	11.808	12.879	14.573	16.151	40.113	43.195	46.963	49.645	55.476
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993	56.892
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336	58.301
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672	59.703
40	20.707	22.164	24.433	26.509	55.758	59.342	63.691	66.766	73.402
50	27.991	29.707	32.357	34.764	67.505	71.420	76.154	79.490	86.661
60	35.534	37.485	40.482	43.188	79.082	83.298	88.379	91.952	99.607
70	43.275	45.442	48.758	51.739	90.531	95.023	100.425	104.215	112.317
80	51.172	53.540	57.153	60.391	101.879	106.629	112.329	116.321	124.839
90	59.196	61.754	65.647	69.126	113.145	118.136	124.116	128.299	137.208
100	67.328	70.065	74.222	77.929	124.342	129.561	135.807	140.169	149.449

F-Distribution

The table reports x given $Pr(X \leq x) = 0.95$ and the degrees of freedom n_1 (numerator) and n_2 (denominator).

	n_1													
n_2	1	2	3	4	5	6	7	8	9	10	20	50	100	∞
1	161	200	216	225	230	234	237	239	241	242	248	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.58	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.70	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.44	4.41	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.75	3.71	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.32	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.02	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.80	2.76	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.64	2.59	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.51	2.46	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.40	2.35	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.31	2.26	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.24	2.19	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.18	2.12	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.28	2.12	2.07	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.23	2.08	2.02	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.19	2.04	1.98	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.16	2.00	1.94	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.12	1.97	1.91	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.10	1.94	1.88	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.07	1.91	1.85	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.05	1.88	1.82	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.03	1.86	1.80	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.01	1.84	1.78	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	1.99	1.82	1.76	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	1.97	1.81	1.74	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	1.96	1.79	1.73	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	1.94	1.77	1.71	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	1.93	1.76	1.70	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.84	1.66	1.59	1.51
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.78	1.60	1.52	1.44
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.75	1.56	1.48	1.39
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.72	1.53	1.45	1.35
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.70	1.51	1.43	1.32
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.69	1.49	1.41	1.30
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.68	1.48	1.39	1.28
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.57	1.35	1.24	1.01